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ANALYTIC EXTENSIONS AND SELECTIONS[1]

BY

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ABSTRACT. Let G be a closed subset of the closed unit disc in C, let F be a closed subset of the unit circle of measure 0 and let Φ map G into the class of all open subsets of a complex Banach space X. Under suitable additional assumptions on Φ we prove that given any continuous function $f: F \to X$ satisfying $f(z) \in \text{closure}(\Phi(z))$ $(z \in F \cap G)$ there exists a continuous function $\tilde{f}$ from the closed unit disc into X, analytic in the open unit disc, which extends f and satisfies $\tilde{f}(z) \in \Phi(z)$ $(z \in G - F)$. This enables us to generalize and sharpen known dominated extension theorems for the disc algebra.

Let p be a real valued positive continuous function on the unit circle T in C and let $F \subset T$ be a closed set of Lebesgue measure zero. Given any continuous function $f: F \to C$ satisfying $|f(s)| < p(s)$ $(s \in F)$ there exists a function $\tilde{f}$ in the disc algebra which extends f and satisfies $|\tilde{f}(t)| < p(t)$ $(t \in T)$. This simple dominated extension theorem is a special case of a more general theorem proved by E. Bishop [1]. See [1]–[3], [6], [7], [10] for such theorems in general spaces of continuous functions and see [7] for the most general dominated extension theorem in the disc algebra.

Writing $\Phi(t) = \{z \in C: |z| < p(t)\}$ $(t \in T)$ the above theorem becomes a selection theorem: Given any continuous function $f: F \to C$ satisfying $f(s) \in \Phi(s)$ $(s \in F)$ there exists a function $\tilde{f}$ in the disc algebra which extends f and satisfies $\tilde{f}(t) \in \Phi(t)$ $(t \in T)$.

In the present paper we use some ideas of [5] to prove a selection theorem for the disc algebra which generalizes and sharpens known results on dominated extensions.

Throughout, we denote by Δ, $\bar{\Delta}$ and $\partial\Delta$ the open unit disc in C, its closure and its boundary, respectively. If X is a complex Banach space and $r > 0$ we write $B_r(X) = \{x \in X: \|x\| < r\}$. Let $x \in X$ and $S, T \subset X$. We write $x + S = \{x + u: u \in S\}$ and $S + T = \{u + v: u \in S, v \in T\}$ and denote by $\bar{S}$ the closure of S. By $A(\Delta, X)$ we denote the Banach space of all continuous functions from $\bar{\Delta}$ to X which are analytic on Δ and by A we denote the disc algebra $A(\Delta, C)$. We write $I = \{t: 0 \leqslant t \leqslant 1\}$ and denote the set of all positive integers by N.

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Suppose that $\{p_\alpha;\ \alpha \in \mathcal{Q}\}$ is a family of nonempty open subsets of X. For each $\alpha \in \mathcal{Q}$ let $x_\alpha \in \overline{P}_\alpha$. We say that the sets P_α $(\alpha \in \mathcal{Q})$ are *equi-locally connected* at the points x_α if given any $\varepsilon > 0$ there is some $\delta > 0$ such that for every $\alpha \in \mathcal{Q}$ the set $(x_\alpha + B_\delta(X)) \cap P_\alpha$ is contained in a connected component of $(x_\alpha + B_\varepsilon(X)) \cap P_\alpha$ [9]. We call any such $\delta(\cdot)$ a *modulus of equi-local connectedness* of the sets P_α $(\alpha \in \mathcal{Q})$ at the points x_α.

Let $S \subset \overline{\Delta}$ be a closed set and let X be a Banach space. We call the *graph* of a map Φ: $S \to 2^X$ the set of all pairs $(z, x) \in S \times X$ such that $x \in \Phi(z)$. We say that Φ is *open* if its graph is open in $S \times X$; equivalently, Φ is open if given any $z \in S$ and any $x \in \Phi(z)$ there is some $\varepsilon > 0$ such that $x + B_\varepsilon(X) \subset \Phi(u)$ $(u \in S\colon |u - z| < \varepsilon)$. In particular, if Φ is open then $\Phi(z)$ is open for every $z \in S$.

Our main result is the following:

THEOREM. *Let X be a complex Banach space and let $G \subset \overline{\Delta}$ be a closed set. Assume that Φ: $G \to 2^X$ is an open map such that $g(z) \in \Phi(z)$ $(z \in G)$ for some $g \in A(\Delta, X)$. Let $F \subset \partial\Delta$ be a closed set of measure* 0 *and let f: $F \to X$ be a continuous function such that $f(s) \in \overline{\Phi(s)}$ $(s \in G \cap F)$. Assume that $\Phi(s)$ is connected for each $s \in G \cap F$ and that the sets $\Phi(s)$ $(s \in G \cap F)$ are equi-locally connected at the points $f(s)$. Then there exists an extension $\tilde{f} \in A(\Delta, X)$ of f which satisfies $\tilde{f}(z) \in \Phi(z)$ $(z \in G - F)$.*

LEMMA. *Under the assumptions of the theorem with $G = \overline{\Delta}$, let $U \subset \overline{\Delta}$ be a neighbourhood of F and let $\varepsilon > 0$. Suppose that $F = \bigcup_{i=1}^m F_i$ where the F_i $(1 \leqslant i \leqslant m)$ are pairwise disjoint nonempty closed sets. Assume that u,v: $F \to X$ are two functions such that $v(s) \in \Phi(s)$ $(s \in F)$ and such that $u|F_i$ and $v|F_i$ are constant for each i $(1 \leqslant i \leqslant m)$. Suppose that there is some $\tilde{u} \in A(\Delta, X)$ which extends u and satisfies $\tilde{u}(z) \in \Phi(z)$ $(z \in \overline{\Delta})$. Then there is an extension $\tilde{v} \in A(\Delta, X)$ of v which satisfies $\tilde{v}(z) \in \Phi(z)$ $(z \in \overline{\Delta})$ and $\|\tilde{u}(z) - \tilde{v}(z)\| < \varepsilon$ $(z \in \overline{\Delta} - U)$. If, in addition, $R > 0$ and*

$$\|u(s) - f(s)\| < \delta(R), \qquad \|v(s) - f(s)\| < \delta(R) \qquad (s \in F)$$

where $\delta(\cdot)$ is a modulus of equi-local connectedness of the sets $\Phi(s)$ $(s \in F)$ at the points $f(s)$ then one may choose $\tilde{v}$ so that $\|\tilde{u} - \tilde{v}\| < 4R$.

PROOF. We prove both assertions; the proof can be easily adapted to prove only the first assertion.

Observe first that by compactness of $\overline{\Delta}$, by the continuity of $\tilde{u}$ and by the fact that Φ is open there is some $\eta > 0$ such that $\tilde{u}(z) + B_\eta(X) \subset \Phi(z)$ $(z \in \overline{\Delta})$. For the moment, fix $s \in F$. The assumptions imply that there is a path p: $I \to \Phi(s)$ such that $p(0) = u(s)$, $p(1) = v(s)$ and $\|p(t) - f(s)\| < R$ $(t \in I)$. Since Φ is open and since $p(I)$ is compact there is some $r' > 0$ such that $p(I) + B_{r'}(X) \subset \Phi(z)$ $(z \in \overline{\Delta}, |z - s| < r')$.

Since u and v are constant on F_i for $1 \leqslant i \leqslant m$ there are paths $p_i\colon I \to X$ $(1 \leqslant i \leqslant m)$ and an $r > 0$ such that

$$p_i(0) = u(s), \qquad p_i(1) = v(s) \qquad (s \in F_i, 1 \leqslant i \leqslant m),$$

$$p_i(I) + B_{3r}(X) \subset \Phi(z) \qquad \left(z \in \bar{\Delta}, \operatorname{dist}(z, F_i) < r, 1 \leqslant i \leqslant m\right),$$

$$\operatorname{diam} p_i(I) < 2R \qquad (1 \leqslant i \leqslant m),$$

and $r < \min\{\varepsilon, R, \eta\}$. By the continuity of $\tilde{u}$ we may choose pairwise disjoint neighbourhoods $U_i \subset U$ of F_i, respectively, such that

$$p_i(I) + B_{3r}(X) \subset \Phi(z), \qquad \|\tilde{u}(z) - p_i(0)\| < r \qquad (z \in U_i, 1 \leqslant i \leqslant m).$$

By [4, Lemma 4] there exists for each i $(1 \leqslant i \leqslant m)$ an $h_i \in A(\Delta, X)$ such that

$$h_i|F_i = (v - u)|F_i \qquad (1 \leqslant i \leqslant m),$$

$$h_i|F_j = 0 \qquad (1 \leqslant j \leqslant m, j \neq i),$$

$$h_i(\bar{\Delta}) \subset -p_i(0) + p_i(I) + B_r(X) \qquad (1 \leqslant i \leqslant m)$$

$$\|h_i(z)\| < r/m \qquad \left(z \in \bar{\Delta} - U_i, 1 \leqslant i \leqslant m\right).$$

Put $\tilde{v} = \tilde{u} + \sum_{i=1}^{m} h_i$. As in [5, p. 375] it is easy to see that $\tilde{v}$ has all the required properties. Q.E.D.

Proof of the Theorem. It suffices to prove the Theorem in the case when $G = \bar{\Delta}$ (otherwise define $\Psi(z) = \Phi(z)$ $(z \in G)$, $\Psi(z) = X$ $(z \in \bar{\Delta} - G)$, observe that Ψ is an open map and apply the theorem to Ψ). Further, it suffices to prove the Theorem in the special case when $g = 0$ (otherwise define $\Psi(z) = -g(z) + \Phi(z)$ $(z \in G)$, observe that, by the continuity of g, Ψ is open, apply the theorem to Ψ and to the function $s \mapsto h(s) = -g(s) + f(s)$ and finally put $\tilde{f} = g + \tilde{h}$). So assume that $G = \bar{\Delta}$ and $g = 0$.

Let $\delta(\cdot)$ be a modulus of equi-local connectedness of $\Phi(s)$ $(s \in F)$ at the points $f(s)$ and let δ_n be a decreasing sequence of positive numbers converging to 0 and satisfying $\delta_n < \delta(\frac{1}{4}\cdot 2^{-n})$ $(n \in N)$. As in the proof of Lemma 5, [5, pp. 371–372] it follows from our assumptions that for each n there is a decomposition $F = \bigcup_{i=1}^{m_n} F_i$ where the F_i are pairwise disjoint nonempty compact sets, and a function $f_n\colon F \to X$ such that $f_n|F_i$ is constant for each i $(1 \leqslant i \leqslant m_n)$ and such that

(a) $f_n(s) \in \Phi(s)$ $(s \in F, n \in N)$,

(b) $\|f_n(s) - f(s)\| < \delta_n$ $(s \in F, n \in N)$.

We may assume that for all n, each element of $(n + 1)$st decomposition is contained in an element of nth decomposition. Since Φ is open, $0 \in \Phi(z)$ for all $z \in \bar{\Delta}$, and since $\bar{\Delta}$ is compact there is some $\varepsilon_0 > 0$ such that $B_{\varepsilon_0}(X) \subset \Phi(z)$ $(z \in \bar{\Delta})$. Let $U_n \subset \bar{\Delta}$ be a decreasing sequence of neighbourhoods of F

such that $F = \bigcap_{n=1}^{\infty} U_n$. Assume that there exist a decreasing sequence ε_n of positive numbers satisfying $\varepsilon_n < \varepsilon_0$ ($n \in N$) and a sequence $g_n \in A(\Delta, X)$, $g_0 = 0$, with the following properties:

(i) $g_n|F = f_n|F$ ($n \in N$),

(ii) $\|g_n - g_{n-1}\| < 1/2^{n-1}$ ($n \in N, n \geq 2$),

(iii) $\|g_n(z) - g_{n-1}(z)\| < \varepsilon_{n-1}/2^n$ ($z \in \bar{\Delta} - U_n, n \in N$),

(iv) $g_n(z) + B_{\varepsilon_n}(X) \subset \Phi(z)$ ($z \in \bar{\Delta}, n \in N$).

By (ii), g_n converge uniformly on $\bar{\Delta}$ so putting $\tilde{f}(z) = \lim_{n\to\infty} g_n(z)$ ($z \in \bar{\Delta}$) we have $\tilde{f} \in A(\Delta, X)$. By (i) and (b) $\tilde{f}|F = f$. Further, let $z \in \bar{\Delta} - U_1$. Writing $\tilde{f}(z) = \sum_{n=0}^{\infty} (g_{n+1}(z) - g_n(z))$ it follows by (iii) that $\tilde{f}(z) \in B_{\varepsilon_0}(X) \subset \Phi(z)$. Finally, if $z \in U_1 - F$ then for some $n \in N$ we have $z \notin U_j$ ($j > n$) so by (iii) and (iv) it follows that

$$\tilde{f}(z) = g_n(z) + \sum_{j=n}^{\infty} \left[g_{j+1}(z) - g_j(z) \right] \in g_n(z) + B_{\varepsilon_n}(X) \subset \Phi(z)$$

and consequently $\tilde{f}$ has all the required properties.

It remains to prove the existence of ε_n and g_n with the above properties. Put $g_0 = 0$. By the first part of the Lemma there is some $g_1 \in A(\Delta, X)$ satisfying $g_1(z) \in \Phi(z)$ ($z \in \bar{\Delta}$) and such that (i) and (iii) hold for $n = 1$. Since Φ is open, $g_1(z) \in \Phi(z)$ for all $z \in \bar{\Delta}$ and since $\bar{\Delta}$ is compact there is some ε_1 with $0 < \varepsilon_1 < \varepsilon_0$ such that $g_1(z) + B_{\varepsilon_1}(X) \subset \Phi(z)$ ($z \in \bar{\Delta}$). Let $m \in N$ and assume that $g_m \in A(\Delta, X)$ satisfies (i) and (iv) for some $\varepsilon_m > 0$. By the Lemma (a) and (b) imply that there is some $g_{m+1} \in A(\Delta, X)$ satisfying (i)–(iii) for $n = m + 1$ and such that $g_{m+1}(z) \in \Phi(z)$ ($z \in \bar{\Delta}$). Again, since Φ is open, $g_{m+1}(z) \in \Phi(z)$ ($z \in \bar{\Delta}$) and since $\bar{\Delta}$ is compact there is some ε_{m+1} with $0 < \varepsilon_{m+1} < \varepsilon_m$ such that $g_{m+1}(z) + B_{\varepsilon_{m+1}}(X) \subset \Phi(z)$ ($z \in \bar{\Delta}$). Q.E.D.

Next we present some simple applications. In Corollaries 1–6 below *G can be either* $\bar{\Delta}$ *or* $\partial\Delta$.

COROLLARY 1. *Let X be a complex Banach space and let* $p\colon G \to (0, \infty)$ *be a lower semicontinuous function. Given any closed set* $F \subset \partial\Delta$ *of measure* 0 *and any continuous function* $f\colon F \to X$ *satisfying* $\|f(s)\| \leq p(s)$ ($s \in F$) *there exists* $\tilde{f} \in A(\Delta, X)$ *which extends f and satisfies* $\|\tilde{f}(z)\| < p(z)$ ($z \in G - F$). *Moreover, if* $z_j \in \Delta$ ($1 \leq j \leq k$) *and if* n_j ($1 \leq j \leq k$) *are positive integers,* $\tilde{f}$ *can be chosen to have a zero at* z_j *of order at least* n_j.

PROOF. Since p is lower semicontinuous the map $z \mapsto \Phi(z) = \{x \in X\colon \|x\| < p(z)\}$ is open on G. Further, since p is lower semicontinuous there is some $\delta > 0$ such that $p(z) \geq \delta$ ($z \in G$). Consequently $g \in A(\Delta, X)$ defined by $g(z) = 0$ ($z \in \bar{\Delta}$) satisfies $g(z) \in \Phi(z)$ ($z \in G$). Further, it is easy to see that any family $\{P_\alpha;\ \alpha \in \mathcal{A}\}$ of nonempty open convex subsets of X is equi-locally connected at the points x_α for any $x_\alpha \in \bar{P}_\alpha$ ($\alpha \in \mathcal{A}$) so the sets

$\Phi(s)$ $(s \in F)$ are equi-locally connected at $f(s)$. Now the first assertion follows by the Theorem. To prove the second assertion, multiply $\tilde{f}$ by $\varphi \in A$ which satisfies $\varphi|F = 1$, $|\varphi(z)| \leqslant 1$ $(z \in \bar{\Delta})$ and has a zero at z_j of order at least n_j [**2**, Theorem, pp. 284–285]. Q.E.D.

Corollary 1 sharpens and generalizes [**2**, Theorem, pp. 284–285]. Note that in the case when $X = C$ Corollary 1 is an easy consequence of [**6**, Theorem 3].

COROLLARY 2. *Let X be a complex Banach space and let $P \subset X$ be a nonempty open connected set which is locally connected at every point of $\bar{P}$. Let φ: $G \to C$ be a continuous function such that $\varphi(z)g(z) \in P$ $(z \in G)$ for some $g \in A(\Delta, X)$. Given any closed set $F \subset \partial\Delta$ of measure 0 and any continuous function f: $F \to X$ satisfying $\varphi(s)f(s) \in \bar{P}$ $(s \in F)$ there exists $\tilde{f} \in A(\Delta, X)$ which extends f and satisfies $\varphi(z)\tilde{f}(z) \in P$ $(z \in G - F)$.*

PROOF. Define $\Phi(z) = \{x \in X\colon \varphi(z)x \in P\}$ $(z \in G)$. Since P is open and since φ is continuous Φ is an open map; since P is connected $\Phi(z)$ is connected for each $z \in G$. By the continuity of φ and f the set $S = \{\varphi(s)f(s),\ s \in F\} \subset \bar{P}$ is compact and consequently P is uniformly locally connected on S [**5**]. Since φ is bounded on F it follows easily that the sets $\Phi(s)$ $(s \in F)$ are equi-locally connected at the points $f(s)$. Now the assertion follows by the Theorem. Q.E.D.

Similarly we prove

COROLLARY 3. *Let X be a complex Banach space and let $P \subset X$ be a nonempty open connected set which is locally connected at every point of $\bar{P}$. Let h: $G \to X$ be a continuous function such that $h(z) + g(z) \subset P$ $(z \in G)$ for some $g \in A(\Delta, X)$. Given any closed set $F \subset \partial\Delta$ of measure 0 and any continuous function f: $F \to X$ satisfying $h(s) + f(s) \in \bar{P}$ $(s \in F)$ there exists $\tilde{f} \in A(\Delta, X)$ which extends f and satisfies $h(z) + \tilde{f}(z) \in P$ $(z \in G - F)$.*

Next we present some dominated extension theorems for the disc algebra.

COROLLARY 4. *Let p: $G \to [0, \infty)$ be an upper semicontinuous (USC) function and let q: $G \to (0, \infty)$ be a lower semicontinuous (LSC) function such that $p(z) < |g(z)| < q(z)$ $(z \in G)$ for some $g \in A$. Given any closed set $F \subset \partial\Delta$ and any continuous function f: $F \to C$ satisfying $p(s) \leqslant |f(s)| \leqslant q(s)$ $(s \in F)$ there is an $\tilde{f} \in A$ which extends f and satisfies $p(z) < |f(z)| < q(z)$ $(z \in G - F)$.*

PROOF. Define $\Phi(z) = \{\zeta \in C\colon p(z) < |\zeta| < q(z)\}$ $(z \in G)$. Let $z_0 \in G$ and let $\zeta_0 \in \Phi(z_0)$. For some $\varepsilon > 0$ we have $p(z_0) + \varepsilon < |\eta| < q(z_0) - \varepsilon$ for all $\eta \in \zeta_0 + B_\varepsilon(C)$. Since p is USC and since q is LSC there is a neighbourhood $U \subset G$ of z_0 such that $p(z) < p(z_0) + \varepsilon$, $q(z) > q(z_0) - \varepsilon$ $(z \in U)$ and consequently Φ is open on G. Clearly $\Phi(z)$ is connected for every $z \in G$. Let $0 \leqslant r < R$ and let $S = \{\zeta \in C: r < |\zeta| < R\}$. It is easy to see that $(z + B_\varepsilon(C)) \cap S$ is connected for every $z \in \bar{S}$ and for every $\varepsilon > 0$. Consequently the sets $\Phi(s)$ $(s \in F)$ are equi-locally connected at the points $f(s)$. Now the assertion follows by the theorem. Q.E.D.

REMARK. To prove Corollary 4 in the case when $G = \partial\Delta$ one needs to assume only that $p(z) < q(z)$ $(z \in G)$ and one does not need the existence of $g \in A$. Namely, by [**10**, Theorem 5.3, p. 15] there exists a continuous function φ: $G \to R$ satisfying $p(z) < \varphi(z) < q(z)$ $(z \in G)$ and since G is compact there is some $\varepsilon > 0$ such that $p(z) + \varepsilon < \varphi(z) < q(z) - \varepsilon$ $(z \in G)$. By [**11**, p. 216] A approximates in modulus on $\partial\Delta$ so there is some $g \in A$ such that $\varphi(z) - \varepsilon < |g(z)| < \varphi(z) + \varepsilon$ $(z \in G)$ and consequently $p(z) < |g(z)| < q(z)$ $(z \in G)$.

COROLLARY 5. *Let p_1, q_1: $G \to R$ be two upper semicontinuous functions and let p_2, q_2: $G \to R$ be two lower semicontinuous functions such that $p_1(z) <$ $\operatorname{Re} g(z) < p_2(z)$, $q_1(z) < \operatorname{Im} g(z) < q_2(z)$ $(z \in G)$ for some $g \in A$. Given any closed set $F \subset \partial\Delta$ of measure 0 and any continuous function f: $F \to C$ satisfying $p_1(s) \leqslant \operatorname{Re} f(s) \leqslant p_2(s)$, $q_1(s) \leqslant \operatorname{Im} f(s) \leqslant q_2(s)$ $(s \in F)$ there exists an $\tilde{f} \in A$ which extends f and satisfies $p_1(z) < \operatorname{Re} \tilde{f}(z) < p_2(z)$, $q_1(z) < \operatorname{Im} \tilde{f}(z) < q_2(z)$ $(z \in G - F)$.*

PROOF. Define $\Phi(z) = \{\zeta \in C$: $p_1(z) < \operatorname{Re} \zeta < p_2(z)$, $q_1(z) < \operatorname{Im} \zeta <$ $q_2(z)$ $(z \in G)$ and observe that by the semicontinuity of p_1, p_2, q_1, q_2 Φ is open on G. Further, since $\Phi(z)$ is convex for every $z \in G$ it follows that the sets $\Phi(s)$ $(s \in F)$ are equi-locally connected at the points $f(s)$. Now the assertion follows by the Theorem. Q.E.D.

COROLLARY 6. *Let p: $G \to (0, \infty)$ be a lower semicontinuous function. Given any closed set $F \subset \partial\Delta$ of measure 0 and any continuous function f: $F \to C$ satisfying $|f(s)| \leqslant p(s)$, $\operatorname{Re} f(s) \geqslant 0$ $(s \in F)$ there exists an $\tilde{f} \in A$ which extends f and satisfies $|\tilde{f}(z)| < p(z)$, $\operatorname{Re} \tilde{f}(z) > 0$ $(z \in G - F)$.*

PROOF. Define $\Phi(z) = \{\zeta \in C: |\zeta| < p(z), \operatorname{Re} \zeta > 0\}$ $(z \in G)$. Since p is LSC, Φ is open on G. Since p is LSC and positive and since G is compact there is some $\delta > 0$ such that $p(z) \geqslant \delta$ $(z \in G)$. Define $g \in A$ by $g(z) =$ $\delta/2$ $(z \in \bar{\Delta})$. Clearly $g(z) \in \Phi(z)$ $(z \in G)$. Since $\Phi(z)$ is convex for every $z \in G$ it follows that the sets $\Phi(s)$ $(s \in F)$ are connected and equi-locally connected at the points $f(s)$. Now the assertion follows by the Theorem. Q.E.D.

Corollary 6 with $G = \partial\Delta$ sharpens [7, Corollary 4.5]. Corollary 6 with $G = \bar{\Delta}$ answers a question in [7, p. 294].

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